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ABSTRACT
In *Metaphysics* Gamma 3 Aristotle declares that the philosopher investigates things that are *qua* things that are and that he therefore should be able to state the firmest principles of everything. The firmest principle of all is identified as the principle of non-contradiction (PNC). The main focus of Gamma 3 is Aristotle's proof for this identification. This paper begins with remarks about Aristotle's notion of the firmness of a principle and then offers an analysis of the firmness proof for PNC. It focuses on some key assumptions of the proof and on the range and force of the proof. Aristotle closes Gamma 3 with the claim that PNC is ultimate in the sense that all other principles somehow rest on it. This, rather controversial, claim is given a defensible reading and shown to be central to the chapter's effort to establish PNC as *the* firmest principle of all. As such it completes the firmness proof and is not simply an appended remark.

Midway through *Metaphysics* Gamma 3 Aristotle announces that it falls to the philosopher to investigate things that are *qua* things that are (περὶ τῶν ὄντων ᾗ ὄντα) and that, as such, he should be able to state the firmest principles of everything. He then offers an account of what it is to be a firmest principle (1005b11-18), and immediately identifies the principle of non-contradiction (PNC) as the firmest principle of all (1005b18-22). The balance of the chapter contains a proof for this identification (1005b22-32), with a closing flourish promoting the principle's ultimacy (1005b32-34). Over the years a number of criticisms have been directed at Aristotle's account, especially at the Indubitability Proof, as I shall call the firmness proof at 1005b22-32. The present paper is concerned mainly with some key assumptions of the proof and aims to restore a measure of credibility to Aristotle's account. I begin, however, with his characterization of a firmest principle.

1. *The Notion of a Firmest Principle*

At 1005b8-11 Aristotle opens discussion of the "firmest principle of all" with a rule-to-case argument designed to link the principle to the study of being in general. Rule and case are given, respectively, as

R. If someone has the best understanding of a genus, then that person can state the firmest principles of that domain (τοῦ πράγματος);

and

C. If someone has the best understanding of things that are *qua* things that are, then that person can state the firmest principles of everything.

It is the philosopher who enjoys the understanding advertised in (C) and so the philosopher who can state the firmest principles of everything. Now for (R) and (C) to constitute a rule-to-case argument, things that are *qua* things that are, or, as I shall sometimes say, being *qua* being, would have to constitute a genus, and the principles of being *qua* being would have to be principles of everything. The latter claim is attested earlier in the chapter when Aristotle points out that the philosopher busies himself with everything that is, not just things that fall into a certain genus distinct from others. So in (C) we are to understand "everything" as "everything that is." In similar manner, being *qua* being will not constitute a 'certain' genus distinct from the others, but rather a ubiquitous genus containing everything, at least, everything that is *qua* thing that is. This, too, is prepared for in Chapter 2, where Aristotle loosens the reins on what counts as a genus.[1] I shall say no more about the rule-to-case argument itself, although much could be said. For, at the moment, the main task is to achieve some clarity on the central notion of the argument, namely, the notion of a firmest principle.

At 1005b11-18 Aristotle tells us what he has in mind by a firmest principle:

> [*i*] A principle about which it is impossible to be in error is firmest (βεβαιοτάτη) of all. For [*ii*] a principle of that kind is necessarily the most intelligible (γνωριμωτάτην), since [*iii*] everyone makes mistakes on matters about which he does not have understanding; and [*iv*] it is non-hypothetical (ἀνυπόθετον), since [*v*] what is necessarily possessed by one who apprehends any of the things-that-are (ἣν γὰρ ἀναγκαῖον ἔχειν τόν ὁτιοῦν ξυνιέτα τῶν ὄντων) is not a hypothesis (τοῦτ οὐχ ὑπόθεσις), and [*vi*] what one necessarily understands who understands

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[1] Lest the reader worry that this contradicts the well entrenched thesis that being is not a genus, Gamma 2, 1004a23-25, allows that a group of items may be the subject of a single discipline either if they have the same formula ("their formulae are connected by a single thing") or if their formulae refer to one thing ("their formulae are connected by reference to one thing"). Being is not a genus of the first sort, but it may well be a genus of the second sort. Both are adequate for establishing a discipline, but only the latter could serve for purposes of a general science of being or 'things that are'. It is the first sort of genus that is proscribed by the entrenched thesis.

anything (ὃ δὲ γνωρίζειν ἀναγκαῖον τῷ ὁτιοῦν γνωρίζοντι) is necessarily part of the equipment he comes with. It is plain, then, that [*vii*] a principle of that kind is firmest of all. (following Kirwan)

By itself (*i*) might appear to be part of a definition of the notion of a firmest principle. But Aristotle immediately supports (*i*) with an argument, so it is best taken as a thesis that needs to be supported or, at the very least, explained. The thesis is this:

A. If (a) error is impossible regarding a principle, *P*, then (b) *P* is firmest.

Thesis (A) ranges over principles and connects a principle's impossibility for error, (a), to its status as firmest, (b). Before moving to the argument for the thesis, a couple of points bear mention.

First, note that, *were* Aristotle claiming that (b) is a sufficient condition for (a), one might be tempted to construe firmness as a metaphysical property that happens to render a proposition immune to error. But it is not, for he clearly means to explain why satisfying (a) confers firmness on a principle. Rather, because (maximal) firmness is a property inherited by a principle on the grounds that no one can err with respect to it, firmness is a kind of doxastic property accruing to a principle because of what believers are and are not capable of with respect to it. Still this is not a matter that is 'up to the believer', since Aristotle's argument for (a) effectively denies believers any choice in the matter and, indeed, as we shall see, is itself based on a metaphysical principle. So the firmness of interest to the argument is an odd sort of doxastic property.

Second, note that (A) neither says nor requires that there be just a single firmest principle, and generally there is no objection to there being several maximally firm principles. So we might take '*P*' as a variable in an eventual canonical formulation of (A). Later in Gamma 3 Aristotle asserts that there is a single principle that is the firmest of all, namely, PNC, and that PNC is basic, that is, that it underlies, or is the principle of, all the other axioms. Eventually, I will say something about both assertions, but for now we need to turn to the argument for (A).

As mentioned, thesis (A) connects a principle's impossibility for error, (a), to its status as firmest, (b). Suppose we first say something about the broad outlines of the argument for the thesis, beginning with the fact it rests on two conditionals that install (a) in the antecedent position. Contained, respectively, in (*ii*) and (*iv*) these are:

1. If (a) error is impossible regarding a principle, *P*, then (c) *P* is necessarily most intelligible,

and

2. If (a) error is impossible regarding a principle, *P*, then (d) *P* is non-hypothetical.

Propositions (1) and (2) trivially yield

3. If (a) error is impossible regarding a principle, *P*, then (c) *P* is necessarily most intelligible, and (d) *P* is non-hypothetical.

Aristotle will give support arguments for both (1) and (2), and we shall get to them in a moment. But, in advance of this, we should note that (3) supports thesis (A) only if we provide, in addition to (1) and (2), an extra premise:

4. If (c) *P* is necessarily most intelligible and (d) *P* is non-hypothetical, then (b) *P* is firmest.

For from (3) and (4) we get

5. If (a) error is impossible regarding a principle, *P*, then (b) *P* is firmest,

which is just thesis (A). Premise (4) is unstated, it is but required by the argument's structure. Its status is unclear (is it part of the very notion or definition of a firmest principle?) and I shall say something about this below. First, however, we need to look at the support arguments for (1) and (2). If nothing else, they contain hints about the content of these premises.

In part (*iii*) of the passage Aristotle begins the support argument for (1) by making failure to understand something a sufficient condition for being mistaken about it. Taken literally, the line yields

1.1. If *x* does not understand *P*, then *x* errs regarding *P*.

As it stands, (1.1) is inadequate for at least two reasons. First, I may not understand *P*, but still, as a matter of luck, I might avoid error with regard to *P*. So (1.1) is arguably false. But, second, even if true, (1.1) is too weak for its intended purpose. For it is equivalent to

1.2. If *x* does not err regarding *P*, then *x* understands *P*,

and (1.2) cannot provide support for (1) because the antecedent of (1) is stronger – asserting that error regarding *P* is impossible. A more successful, 'rectified', support argument can be gotten by replacing (1.1) with

1.1*. If it is possible that *x* does not understand *P*, then it is possible that *x* errs regarding *P*.

Proposition (1.1*) appears to be true, or at least plausible. Moreover, it is equivalent to

> 1.2*. If (a) x cannot err regarding P, then (e) x necessarily understands P,

which does yield (1), so long as we also assume

> 1.3*. If (e) x necessarily understands P, then (c) P is necessarily most intelligible.

So the rectified support argument for (1) appears to run smoothly. From (1.2*): if (a), then (e), and (1.3*): if (e), then (c), we infer (1): if (a), then (c).

But there are some worries. In (1.3*), for example, one wonders why the consequent does not read simply "P is necessarily intelligible," rather than "P is necessarily most intelligible." Apart from the fact that (1.3*)'s second 'necessary' probably signals necessity of the inference rather than of what is inferred, it is unclear what it even means to say that a given proposition is 'necessarily intelligible' – other than that it can't fail to be understood whenever entertained. But this would presumably hold of a wide range of propositions and so would not strengthen an argument that is meant to establish conditions on the *firmest* of all principles. As for the notion of a most intelligible proposition, this, too, is not transparent. But at least this notion has some Aristotelian credentials. Witness his insistence that the premises of a demonstration are better known than the conclusion. This is, of course, a relative notion – a first thing is always better known relative to a second. So for P to be the most intelligible principle would, on this account, entail that it is more intelligible than any other principle and that there is no principle as intelligible as it. At the end of Gamma 3, Aristotle suggests that this may, indeed, be the case with the principle of non-contradiction. We shall look at this in Section 8, but for now we are still tracking Aristotle's account of a firmest principle. Let us then turn to the support argument for (2)

This argument, the support argument for (2), is contained in (*v*) and (*vi*) of our passage. The lead premise of the argument is located in (*vi*), which contains the difficult phrase, in Kirwan's translation, "what one necessarily understands who understands anything . . ." The phrase lends itself to at least two rather different readings: (*a*) "what is presupposed by anything anyone understands," and (*b*) "what one understands when anything [less understandable] is understood." The proposed requirement is that if x is more understandable than y, then anyone understanding y will understand *x*. Now reading (*b*) appears to suggest that because checkers is more understandable than quantum field theory, anyone understanding quantum field theory will understand checkers. This is obviously false. One might object that the example turns on Aristotle's notion of being more understandable to us, by which checkers arguably *is* more understandable than quantum field theory, while quantum field theory is more understandable *by nature*. But this helps little, for surely it is false that anyone who understands checkers will also understand quantum field theory. The mistake, in both cases, is remedied by requiring that there be some logical or conceptual connection between the items in question. This is preserved in reading (*a*). Thus, for example, geometry is more understandable than optics, for anyone understanding optics must understand geometry but not vice versa. So the premise to be extracted from (*vi*) is this:

> 2.1. If (f) *x*'s understanding anything presupposes *x*'s understanding *P*, then (g) *x* must already have *P* [". . . . is part of his equipment. . . ."].

The second premise of the support argument for (2) is drawn from (*v*) of the text:

> 2.2. If (g) *x* must already have *P* [". . . is part of his equipment. . . ."], then (d) *P* is non-hypothetical.

From (2.1) and (2.2) we can get

> 2.3. If (f) *x*'s understanding anything presupposes *x*'s understanding *P*, then (d) *P* is non-hypothetical,

whose consequent does mention *P*'s non-hypothetical status. But to support (2), this must be shown to follow from *P*'s status as an error-immune principle. This requires an additional premise:

> 2.4. If (a) error is impossible regarding a principle *P*, then (f) *x*'s understanding anything presupposes *x*'s understanding *P*.

Armed with (2.4), which as Kirwan notes is not contained in the text, Aristotle can conclude that a principle's immunity to error entails its non-hypothetical status. This, of course, is just (2), and so (2.4) completes the support argument for (2).

Granting that it is 'formally' complete, the force of the support argument for (2) remains unclear. We also need to assess the acceptability of the crucial, added premise, (2.4), and to determine what is meant by saying that a principle is non-hypothetical.

With respect to the last point, Ross (1924, Vol. 1) held that 'non-hypothetical' (ἀνυπόθετον) appears in 1005b14 "quite in the Platonic sense of the word" and is not meant to oppose a more technical Aristotelian notion of hypothesis, according to which hypotheses assume the existence of the primary subject matter of a science. Such an opposition would confer non-hypothetical status on propositions that did not assume the existence of the subject of a science – hardly a plausible reading for our Gamma 3 passage. As for the Platonic sense of 'non-hypothetical', we may begin with the so-called hypothetical method in the *Meno*. Here a first proposition is explained by introducing a second, a hypothesis, that entails it in an appropriate way, and the second may then be explained in similar fashion by adducing a third proposition as another hypothesis. Insofar as a hypothesis is introduced simply on the strength of its entailments, it enjoys no independent claim to truth. *Republic* 511b remarks that this use of hypotheses cannot lead upwards to a first principle. This yields a weak notion of a non-hypothetical proposition, according to which a non-hypothetical proposition is one that is not assumed for purposes of derivation. However, Plato goes on to add that the mind itself is able to go beyond such hypotheses to what in Grube's rendering is called "the unhypothetical first principle of everything" (τοῦ ἀνυποθέτου ἐπὶ τὴν τοῦ παντός ἀρχὴν, 511b6-7). This notion of a non-hypothetical principle is quite robust, combining two notions – that of a first principle of everything and that of being grasped directly by the mind. Unfortunately, Plato says almost nothing about how the mind comes to do the grasping in question.[2]

Aristotle's idea in (2.3) is slightly different – anything that is presupposed by anything (else) that we understand must be non-hypothetical. For suppose it were not; then there would be something presupposed by it, from which it could be derived, and, hence, it would not be presupposed by everything. This Aristotelian notion strikes me as closer to the weak, rather than the strong, Platonic notion of the non-hypothetical. Whether this Aristotelian sense of ἀνυπόθετον is Platonic or not, something close to it is suggested in *Posterior Analytics* A 10, 76b23-34, by Aristotle's characterization of a hypothesis as what is provable but accepted without proof. By this account, favored by Kirwan,[3] p would be non-hypothetical, if it is not the case that p is provable and accepted without proof. So, by De Morgan's law, p is non-hypothetical if p is not provable or p is not

[2] In this paragraph, I am indebted to conversation with Alan Code.
[3] Kirwan, 1971, 88.

accepted without proof. Thus, there appear to be two kinds of non-hypothetical propositions: those accepted with a proof, i.e., theorems, and those that are not provable. Only the latter case is relevant to principles of the sort mentioned in (2.3). So to call a principle *P* non-hypothetical is just to say that it is not provable. Not only does this agree nicely with (2.3) but also just such a constraint is at work in Gamma 4 where Aristotle denies the possibility of providing a demonstration for PNC – his candidate for the firmest principle.

What, then, of the crucial added premise in the support argument for (2)? In a word, why should we grant (2.4)? Why should the impossibility of error have anything to do with a principle's being presupposed by everything anyone understands? It is important to bear in mind that Aristotle's claim concerns *principles*. Otherwise, one might insist, the proposition, *that it is not the case that Smith is blond and not blond*, would have to be presupposed by anything anyone understands. For, as Aristotle can argue in the final section of Gamma 3, because the proposition is an instance of PNC, error is impossible with regard to it. Even constraining (2.4) to principles, why should we count it true? The worry is, perhaps, clearer from the following formulation, which is equivalent to (2.4):

2.4*. If (f*) *x* understands something, *q*, and *x* does not understand *P*, then (a*) error is possible regarding a principle *P*.

Smith, who understands *q*, simply may not have entertained *P*. Surely, then, he cannot be said to understand *P*; but, just as surely, *P* may be such that he *could not* err with respect to it *were* he to entertain it. Kirwan suggests that (2.4) assumes that where error is impossible so are ignorance and confusion. But this only relocates our question. Why if ignorance and confusion are impossible with respect to *P*, should it follow that *P* is understood by anyone who understands anything? It is, in short, hard to see why we should accept (2.4).[4]

[4] One might hold that because *P* is immune to error, it *cannot* be false and, hence, can only be true. Because of this, *P* is 'entailed' by everything. That is, because *P* is true, $(q \supset P)$ holds for all *q*, false as well as true. In favor of this, one might urge that (2.4) makes its doubt-resistant *P* a presupposition of *understanding*, quite generally, anything at all. As such it would be natural for the thesis to cover both true and false propositions. But in the present context 'understanding' clearly carries epistemic force – to understand (γνωρίζειν) is have some sort of grasp of what something *is*. As Kirwan (1971, 88) points out the verb's basic meaning is to make intelligible. Where *q* is false, grasping it does not make anything intelligible. Here, of course, it is presumed that Aristotle has no interest in the notion of making the *meaning* of a

Some might try for a more acceptable variant on (2.4) by appealing to an alternative reading of (*vi*) of the text. According to this reading, marked '(*b*)' four paragraphs back, (2.4) would say something like

2.4.' If (a) error is impossible regarding a principle *P*, then (f') *x* understands *P* if *x* understands anything less understandable than *P*.

The idea here is that if you can't be mistaken about something, then it is the most understandable of the things you understand. At the very least you understand *P*. Now (2.4') sounds better because it appears to say that if you can't be mistaken about something, then it is the most understandable of the things you understand. But appearances deceive. Why should the fact that *P* is immune to error, by itself, entail that *P* is the most understandable of the things you understand? The antecedent, (a), gives no reason to think that *P* is even understood; and as a reading of (*vi*) the consequent, f', is no more plausible than before, unless *P* is presupposed by anything less understandable than it. But this would make (a) entirely irrelevant. Plus, the argument for (2) now contains (f) in (2.1) and (f') in (2.4'), and this equivocation vitiates the argument completely. Finally, 2.4' bears an uncanny resemblance to (1.3*), a crucial premise in the support argument for (1), and this fact undercuts the independence of the two support arguments. We are, then, I think, obliged simply to retain (2.4) as the 'added' premise in the support argument for (2).

As we have seen, the support argument for (2) claims to establish that a principle immune to error is non-hypothetical. By our account three paragraphs back, non-hypothetical status carries no presumption of knowledge. It simply constrains the non-hypothetical to what is not provable. But the principles of interest to Aristotle are, or at least are akin to, principles of science, and, therefore, they cannot be devoid of epistemic force. For this reason, Aristotle gives an *independent* argument for a principle's being *most intelligible*. This is the support argument for (1). With the two arguments in place, Aristotle infers (5) and, with that, concludes that thesis (A) has been established: If (a) error is impossible regarding a principle, *P*, then (b) *P* is firmest. So *P* merits status as firmest if there can be no principle more intelligible than it and it is more intelligible than all other principles, (c), and if it is not provable, i.e., if there is no principle

proposition intelligible, a notion that would apply to false as well as true propositions. The disinterest is clear from section (*v*) of the text, where apprehending, used synonymously with understanding, is about *things that are* (τῶν ὄντων).

from which it may be proved, (d). Both (c) and (d), the proximate sufficient conditions for firmness, are inherited by any principle that is immune to error. For this reason, Aristotle is free to restrict himself to immunity to error as the qualifying condition for a firmest principle. He does just this in the Indubitability Proof, as we shall shortly see.

2. *The Firmest Principle Stated*

Having explained, apparently to his satisfaction, what is meant by a firmest principle, Aristotle immediately identifies the principle:

> We have then to state next what this principle is: For the same thing to hold and not to hold of the same thing at the same time and in the same respect is impossible, given any further specifications added to guard against dialectical objections (1005b18-20).

This is Aristotle's classic formulation of the principle of non-contradiction. It is a modal proposition, declaring that it is *not possible* for something to have a property and not to have it at the same time and in the same respect:

> 6. It is not possible that there is something, x, such that x has a property, F, and x does not have F.

We may view (6) as an ontological version of PNC insofar as it ranges over things and their properties, rather than over statements or propositions. Somewhat more formally, this difference may be reflected by distinguishing

> 6a. $\neg\Diamond(\exists x)(Fx \wedge \neg Fx)$,

which represents an ontological thesis, from

> 6b. $\neg\Diamond(\exists p)(p \wedge \neg p)$,

which might be termed a logical version of the principle of non-contradiction. Of course, one can always construct a statement corresponding to the fact that a given thing has a certain property, so the ontological version can be made to accord with the logical version. But the logical version of PNC is entirely general and includes statements that cannot be reduced to statements predicating properties of things. The more general version may be attested in Gamma 4, at 1006a1, where Aristotle speaks simply of something's "being and not being." However, as Kirwan points out,[5] the phrase may be elliptical for "being so-and-so and not so-and-so," and thus may range over properties and things. There are, of course, other passages which might be thought to invite a logical reading. In *De Interpretatione* 6, for example, Aristotle says that he will "call an affirmation and a negation which are opposite a contradiction." However, he goes on to add that statements are opposites when they affirm and deny the same thing of the same thing, and this is less general than the logical formulation. In any case, as we shall see, it is the ontological version that Aristotle deploys in Gamma 3's Indubitability Proof. This, plus the fact that in *Metaphysics* Gamma PNC functions as a principle of *things that are*, makes it clear that the ontological version is the preferred formulation for Aristotle's metaphysical purposes.

3. *Proving that PNC is the Firmest Principle: The Indubitability Proof*

Immediately after identifying PNC as the firmest principle, Aristotle remarks that it is the firmest principle "because it fits the specification stated." The specification he has in mind is just the immunity to error that figures in the lead thesis, (A). So we may regard his strategy as follows. Having established the lead thesis, he now moves to show that (a), the antecedent of (A), is satisfied by PNC. Thus, the Indubitability Proof will demonstrate that it is impossible to err with respect to PNC. One would expect this to establish that PNC is *an* instance of *P*, that is, that it is *a* firmest principle. But Aristotle appears to urge something stronger, namely, that PNC is the only instance of *P*, that is, that it is firmer than any other principle and so is *the* firmest principle. This claim is not trouble-free and I shall say something about it later, when I turn to Aristotle's closing flourish to Gamma 3 where PNC is declared the principle of principles and the principle every demonstration ultimately depends on. This ultimacy claim, as I shall call it, is tagged onto the Indubitability Proof, and, although the claim is tied to the immunity to error registered in the antecedent of (A), insofar as it aspires to establish PNC as firmer than all other principles, the ultimacy claim goes beyond (A) itself. Before engaging this issue, in Section 8 below, we must take a look at the Indubitability Proof itself.

The Indubitability Proof is given in ten lines of text:

> [*viii*] This, then, is the firmest of all principles, for [*ix*] it fits the specification stated. For [*x*] it is impossible for anyone to believe that the same thing is and

[5] Kirwan, 1971, 89.

> is not, as some consider Heraclitus said – for it is not necessary that what one says one must also believe. But if [*xi*] it is not possible for contraries to hold good of the same thing simultaneously . . . and if [*xii*] the opinion contrary to an opinion is that of the contradictory, then [*xiii*] obviously it is impossible for the same person to believe simultaneously that the same thing is and is not. For [*xiv*] anyone who made that error would be holding contrary opinions simultaneously. (1005b22-32, following Kirwan 1971)

Aristotle begins, in (*viii*) and (*ix*) by suggesting a link to the lead thesis (A), the thesis connecting immunity to error with maximal firmness. That thesis spoke simply of a principle's being immune to error (see [*i*] of the text, 1005b11-18, above). But it is persons who resist and succumb to error, and so it would seem that a principle is immune to error just in case it is impossible for some*one* to err with respect to it.

So Aristotle is assuming, reasonably, something like

7. If (g) for all x it is impossible that x err with respect to a principle, P, then (a) error is impossible regarding P.

With (7) the terms of argument shift to what individual agents can and cannot do, and this sets the stage for the Indubitability Proof proper. For by raising the question of what sort of proposition or principle can satisfy (g)'s strong condition, (7) invites an answer in terms of what persons can and cannot believe, and, appropriately, Aristotle's answer in (*x*) uses the notion of belief.

Now one might suggest, quite independently of what Aristotle says, that the connection between error and belief is just that if it is impossible for someone to believe something, then it is impossible for that person to err about it. If this, or perhaps its corresponding equivalence, were in the immediate background of Aristotle's argument, he would be assuming

8. For all x, if (h) it is impossible that x believes P, then (a) it is impossible that x errs with respect to P.

But, given (7), which Aristotle must hold, to establish (h) would be to establish that error is impossible with respect to P. Where P is an instance of the negation of PNC, to take the case at hand, Aristotle would be saying that it is impossible to err with respect to, say, "$fa \wedge \neg fa$." Now there is a sense in which one cannot err with respect to instances of the negation of PNC. They are, each and every one, false. And, indeed, one might say the same for any proposition that, in some suitable sense, it was impossible to believe, say, the proposition *that no bachelors are unmarried*. But, for Aristotle, immunity to err conveys firmness, and, surely, it is absurd to confer firmness, let alone the maximal firmness, on these kinds of propositions.[6] Plus, as we saw in the first section, the route to maximal firmness runs through non-hypotheticality and intelligibility. What cannot be believed at all can hardly be counted as an intelligible item.

Rather than (8), then, the assumption in the immediate background of the Indubitability Proof is

> 8*. For all x, if (h) it is impossible that x believes $\neg P$, then (a) it is impossible that x errs with respect to P.

According to (8*), if it is impossible to believe the *negation* of P, it is impossible to err with respect to P. On this account, for me to err about the proposition that the cat is on the mat, it must be possible for me to believe that the cat is not on the mat, or at least to believe something that entails this. At first glance, this seems plausible enough. Plus, it avoids the unsavory result of awarding firmness to instances of the negation of PNC. Pretty clearly, then, Aristotle's strategy is two-fold. First, he establishes the lead thesis (A); and, then, he claims that the antecedent of (A) is satisfied by a select P, arguing that it is impossible for anyone to believe the negation of that P. Of course, the select P is just PNC *or* instances of it – Aristotle does not say whether the target of the Indubitability Proof is PNC itself as opposed to instances of PNC. Settling this issue, which is of some importance, will depend on how the proof unfolds.

What Aristotle says, in (*viii*) – (*ix*), is that PNC is the firmest principle because it is immune to error, for the reason, given in (x), that it is impossible for someone to believe that the same thing is and is not. I shall assume that (*ix*) says that PNC, the principle itself, is what is firmest. This is probably unexceptionable, since the entire discussion proceeds from presumptions about the highest principles of any given science and aims to identify the highest principle or principles of the science of things that are.

[6] I note that Aristotle says nothing about the idea that what one cannot believe, one cannot be in error about. This is a strengthened version of the idea that if I don't believe something, I can hardly be in error about it. This is not an utterly implausible idea. It is, however, an odd sort of immunity to error – one might call it immunity by oversight. Aristotle is after a notion of immunity that is conferred on propositions that one entertains, or at least takes oneself to be entertaining. Still, it would be desirable to have a principled account of the relation between belief and error – something that is not forthcoming from Aristotle, at any rate certainly not in Gamma 3.

So (*ix*) says that PNC is the firmest principle because it is impossible to be in error with respect to it. There are, however, two ways to take this, each attested in the text explaining the notion of a firmest principle. Taken just with (*iii*), (*ix*) would say that PNC is firmest because it is impossible to make mistakes on matters that are governed by it. This allows that Aristotle is claiming that PNC is firmest because it is impossible to make mistakes about instances of it. Taken with (*i*), on the other hand, it looks as if Aristotle ties PNC's firmness to the fact that *it*, the principle itself, is immune to error. It may be that a principle's immunity to error just amounts to the fact that all instances of it are immune to error. Still, the discussion of firmness at 1005b8-18 ends with Aristotle reiterating that it is the principle that is firm. So I shall, at least for the moment, take (*ix*) to say that it is the principle of non-contradiction, itself, about which there can be no error. As already said, this is the specification that PNC must fit to qualify as the firmest principle, and showing that it fits the specification is the task of the Indubitability Proof proper.

What, then, does the proof target – the principle itself or its instances? Assuming that Aristotle means immunity to error to be a property of PNC itself, the Indubitability Proof will establish this either by showing that it is impossible to believe an instance of the negation of PNC or by showing that it is impossible to believe the negation of PNC itself. Formulating the principle as (6a) above, the first option has the Indubitability Proof targeting

9. $\neg\Diamond(\exists x)(\exists z)(x \textbf{ bel } (Fz \wedge \neg Fz))$

as the conclusion, while the second takes the target of the proof to be

10. $\neg\Diamond(\exists x)(x \textbf{ bel } \Diamond(\exists z)(Fz \wedge \neg Fz))$.

The lead-in to the Indubitability Proof does nothing to resolve this. In (*x*) the announced aim of the proof, that it is impossible for anyone to believe that the same thing is and is not (such-and-such), is followed by the rider, "as some consider Heraclitus said." If the example constrains interpretation, then the impossibility in question may concern particular beliefs of the sort Heraclitus supposedly entertained. This suggests that the Indubitability Proof targets beliefs about *instances* of the negation of PNC and so favors something like (9) as the conclusion of the proof. Or the impossibility in question may concern belief in the negation of the principle itself. Here it would be the indubitability of PNC itself that is established. Thus, it would give us (10), with the reference to Heraclitus merely providing an example of the sort of thing that is proscribed by the proof's conclusion. To determine which of these is supported by the Indubitability Proof will require at least an informal statement of the proof.[7]

In proving that one cannot believe the negation of PNC, the Indubitability Proof evidently uses PNC. Adverting to the proof, the opening of Gamma 4 declares, "we have just accepted that it is impossible to be and not be simultaneously, and by means of this we have shown that it is the firmest of all principles."[8] In our Gamma 3 passage, however, it is not clear where PNC enters the discussion. The only explicit mention of contradiction occurs in (*xii*), but there the question concerns which belief is the contrary – the belief of the contrary or the belief of the contradictory. This is an important question, which will occupy us at some length later. For now it is enough to point out that neither this question nor any answer to it bears on introducing PNC itself into the argument. Yet in Gamma 4 Aristotle insists that PNC is crucial to the Indubitability Proof. If so, it will have to be the ontological version, and thus I enter the principle, already given in (6a), as a fresh premise:

11. $\neg\Diamond(\exists x)(Fx \wedge \neg Fx)$.

Lest there be worry on the point, it may be useful to recall that there is nothing illegitimate about using PNC to prove something about PNC. For what is proved is not PNC but a different proposition *about* it, namely, that its negation cannot be believed. Indeed, the result is significant, given the doxastic cast of the proposition proved, for the mere fact that a given state of affairs is implausible, odd, untenable, or even impossible hardly seems sufficient to disqualify or otherwise impugn the possibility of *believing* that the state of affairs obtains. What Aristotle aims to show, against this, is that when it comes to a contradiction, belief in the alleged state of affairs is indeed impugned.

If the text does not explicitly invoke PNC, it does deploy in (*xi*) the companion principle that *contraries* cannot hold of the same thing simultaneously. Ontologically cast, we may formulate this as

12. $\neg\Diamond(\exists x)(Fx \wedge F^*x)$,

[7] More complicated formal treatments are available. See, for example, the analysis in Barnes 1969. For the immediate purpose of isolating the troubling premises that interest me, such informality comes at no cost. A summary of my version of the proof is contained in the Appendix.

[8] So Kirwan 1971.

where F^* is the contrary of F. Although parading in Gamma 3 as an independent point, (12) can be thought of as following from PNC by a principle relating contraries and contradictories. Thus, where F^* is the contrary of F, we have

13. $(x)(F^*x \rightarrow \neg Fx)$,

which says that if something has the property contrary to F, then it has the contradictory of F, or, better, that it simply does not have F (I shall not worry about this difference here). Premise (13) is not explicit in Gamma 3. But without (13), it may be difficult to make sense of Gamma 4's claim that the firmness of PNC is shown by means of PNC itself. Plus, the principle is expressly stated in Gamma 6, at 1011b15-21. So we have reason to take (13) as a premise and (12) as a theorem derived from it and (11).

The Indubitability Proof concerns constraints on what one can believe, in particular, constraints *against* the possibility of believing contradictory states of affairs. So Aristotle needs a principle that relates belief to objects of belief in such a way as to explain why such beliefs are impossible. In (*xii*) he supplies this by remarking that the belief that is contrary to the belief, for instance, that fa is the belief of the contradictory, that is, the belief that $\neg fa$. This might be taken to mean that when Socrates believes that Simmias is not short, he is doing something that is the contrary of believing that Simmias is short. Generalizing, we get something like

14. (x)(x *believes Fa* is contrary to x *believes* $\neg Fa$),

and, agreeing with Aristotle's characterization in *De Interpretatione*, this means that both of the italicized propositions may be false together but not true together.

But the Indubitability Proof must relate those propositions, whose joint belief yields the contrary beliefs of (14), to (12)'s logical rule proscribing possession of contrary *properties*. In order for this to be principled proscription, Aristotle must require that in general believing something involves attribution of a property to the believer, and that such a property is possessed much as any standard property is possessed by a subject. According to Property Attribution, as I shall call this, when Socrates believes that Simmias is short, he has a doxastic property corresponding to the belief, and likewise when he believes that Simmias is not short. Suppose we indicate complex doxastic properties with uppercase B, adding a star, '*', to indicate that the property is the contrary of the enclosed property. Then, [*B*:*Simmias is short*] corresponds to the property, *believing that*

Simmias is short, and [*B*:*Simmias is short*]* represents the contrary of this. Generalizing, for 'affirmative' beliefs we get

14a. $(x)(x\ \mathbf{bel}\ Fa \rightarrow [B{:}Fa]x)$,

and, for the opposed case,

14b. $(x)(x\ \mathbf{bel}\ \neg Fa \rightarrow [B{:}Fa]^{*}x)$.

According to (14b), if someone believes that *a* is not *F*, then by Property Attribution that person has a property that is the *contrary* of the property had by someone who believes that *a* is *F*. Aristotle is not quite precise about this. Should we take the contrary of *believing Fa* to be *believing* $\neg F$, *disbelieving Fa*, or something else? *De Interpretatione* 14 suggests the former, and, as we have seen, (*xii*) of our Gamma 3 passage appears to follow this. In any case, the property contrary to *believing Fa* cannot be *not believing Fa*, for one could have this property without having any doxastic attitude toward *a*, let alone toward *a*'s being *F*. But the proof concerns the possibility of someone's believing *both* a proposition and its negation.

The argument's climax is reached in (*xiii*), when Aristotle announces that it is impossible for someone to believe that the same thing is and is not, because, he explains in (*xiv*), such a person would be holding contrary beliefs simultaneously. I shall construe this, with Barnes 1969, as requiring a premise that licenses simplification for beliefs. This is the general principle

15. $(x)(x\ \mathbf{bel}\ (p \wedge q) \rightarrow x\ \mathbf{bel}\ p \wedge x\ \mathbf{bel}\ q)$,

which says that if someone believes, jointly, a number of propositions, then he believes each severally. Principle (15), which I shall sometimes call Doxastic Simplification, is then applied to the case of interest for the Indubitability Proof, namely, belief in contradictory propositions. This gives us the premise that is relevant to the argument, namely:

15a. $(x)(x\ \mathbf{bel}\ (Fa \wedge \neg Fa) \rightarrow x\ \mathbf{bel}\ Fa \wedge x\ \mathbf{bel}\ \neg Fa)$.

The antecedent of (15a) introduces the offending belief, namely, belief in contradictory states of affairs. The argument against this proceeds by combining (15a) with (14a) and (14b) to get, respectively,

16a. $(x)(x\ \mathbf{bel}\ (Fa \wedge \neg Fa) \rightarrow [B{:}Fa]x)$,

and

16b. $(x)(x\ \mathbf{bel}\ (Fa \wedge \neg Fa) \rightarrow [B{:}Fa]^{*}x)$.

These, in turn, trivially yield

17. $(x)(x\ \mathbf{bel}\ (Fa \wedge \neg Fa) \rightarrow [B{:}Fa\ ]x \wedge [B{:}Fa]^{*}x)$.

Intuitively, the argument is this. Were Antiphon to believe that *Fa and* $\neg$*Fa*, he would believe, separately, each of the conjoined beliefs. Then, thanks to Property Attribution, corresponding to his belief that *Fa*, he will have a certain doxastic property, and likewise for his belief that $\neg$*Fa*. But these properties are contraries, and so his original belief that *Fa and* $\neg$*Fa* entails that contrary properties hold of Antiphon himself. This is just what (17) says.

But by (13) the contrary of a property is also its contradictory. That is, if a thing has the contrary of a property, then it does not have the property. So, in the case at hand, substituting '$[B{:}Fa]^{*}$' for 'F^{*}', we get from (17) and (13)

18. $(x)(x\ \mathbf{bel}\ (Fa \wedge \neg Fa) \rightarrow [B{:}Fa\ ]x \wedge \neg[B{:}Fa]x)$.

In short, anyone who believes *Fa* and $\neg$*Fa* has the corresponding doxastic property, [*B*:*Fa*], and does not have it. And this is a straightforward violation of PNC. So if, arguably with Aristotle, we assume PNC as in (11), the consequent of (18) must be rejected and, hence, we may conclude

19. $(x)\neg(x\ \mathbf{bel}\ (Fa \wedge \neg Fa))$.

According to (19) everyone is such that they do not believe that something has a property and does not have it, and this is just equivalent to

19a. $\neg(\exists x)(x\ \mathbf{bel}\ (Fa \wedge \neg Fa))$.

Presumably, *Fa* is arbitrary and so (19a) yields the conclusion that no one believes that something has a property and does not have it. And because PNC declares the consequent of (18) *impossible*, (19a) can be strengthened to deny the *possibility* of someone's holding such a belief – just as (9) above calls for. Thus, the Indubitablity Proof establishes that contradictions *cannot* be believed and, hence, by (8*) above, that negations of contradictions (i.e., instances of PNC) are immune to error.[9]

[9] For convenience, a summary of my version of the Indubitability Proof is included in the Appendix.

4. *Two Structural Worries about the Indubitability Proof*

The above result would appear to complete the argument for the firmness of the principle of non-contradiction. For it appears to establish that PNC satisfies (a), the antecedent of thesis (A). There are, however, some worries. Chief among these is whether the Indubitability Proof establishes the firmness of PNC, the principle itself, or only the firmness of *instances* of the principle. I shall get to this in the next section. At the moment we need to look at some worries about the proof itself.[10]

Let me begin with a question about the structure of the proof, namely, how exactly (19)/(19a) is derived. Above we took (19) to follow from (11) and (18). However, in this case (12), expressly mentioned in (*xi*) of the text, is by-passed in favor of the direct use of PNC, which, as I have noted, is not expressly mentioned. On the other hand, (19) also follows from (12) and (17), without appealing to (18). Here one might take (12) to be a stand-alone premise, and (11) to be denied an explicit role in the argument. More importantly, perhaps, omission of (18) suggests that, at least in the argument, Aristotle does not explicitly bring doxastic properties under premise (13), which licenses moving from contraries to contradictories. Since (13) does appear in the text, it must play some role in the argument. It seems to me that (13) will have to figure as an assumption needed to get (12), the analogue to PNC for contraries. Since the other

[10] I do not, however, share Priest's worry that the Indubitability Proof is a "hopeless argument" (a) because "it will work only if it is impossible to have violations of the LNC [= PNC] of a certain form, and (b) because "more importantly, it begs the question against someone who claims that they believe contradictions." Objection (a) can be set aside because, dialetheists such as Priest notwithstanding, the question is whether one can *believe* such contradictions, and in any case this would not be established by the fact that contradictions are impossible. Explaining worry (b), Aristotle's reasoning is represented as follows: "If someone believes $\varphi \wedge \neg\varphi$ then they believe φ and they believe $\neg\varphi$; but if they believe $\neg\varphi$ then they don't believe φ (believing φ and believing $\neg\varphi$ are contraries). Hence it follows that they both believe and do not believe φ – a violation of LNC," Priest 1998, 94. But, says Priest, this begs the question because the opponent "will not accede to the claim that believing φ and believing $\neg\varphi$ are contraries." First, this misrepresents Aristotle's reasoning. Aristotle does not simply move from the claim that believing φ and believing $\neg\varphi$ are contraries ([14] of the proof) to the claim that believing the second is not believing the first, and, hence, such a person believes and does not believe φ. Rather, Aristotle uses what I have called Property Attribution, which assigns actual doxastic properties to believing subjects and which does so for all cases of believing. Although one might well quarrel with this idea, it is entirely general and so in no way begs the question. Thus, nothing need prevent the opponent from agreeing that believing φ and believing $\neg\varphi$ are contraries. At issue, rather, is whether it is *possible* for him to hold both beliefs.

assumption needed to get (12) is just (11), this second reading still gives PNC a role in the argument, but only as an implicit assumption. The fact that the text of the Indubitability Proof does not actually contain a statement of PNC, plus the fact that Aristotle opens Gamma 4 by announcing that the proof's conclusion was reached "by means of" PNC, favors this second reading. So it appears that PNC provides the *reason* for rejecting the possibility of joint possession of a property and its contrary. In short, (11) explains *why* (12) holds.[11] In any case, whether it occurs explicitly or provides the grounds for what does occur explicitly, Aristotle feels entitled in Gamma 4 to declare its role in the Indubitability Proof.[12]

A second concern is suggested by Code's remark that for Aristotle a contradiction is not a single conjunctive proposition of the form, '$Fa \wedge \neg Fa$', but rather two propositions: an affirmation, 'Fa', and its opposed negation, '$\neg Fa$'.[13] Aristotle's remark at *De Interpretatione* 17a33-34, "Let us call an affirmation and a negation that are opposite a contradiction," is taken to say that for Aristotle a contradiction (ἀντίφασις) is a pair of opposed statements (ἀντικείμενα), one (the affirmation or κατάφασις) affirming of a subject precisely what the other (the denial or ἀπόφασις) denies of it. Consequently, were someone to believe a contradiction, the object of his belief would not be a single conjunctive proposition; rather he would have "two separate beliefs." One of these will correspond to the affirmation, Fa, and one to the denial, $\neg Fa$. Code observes that this will have consequences for the interpretation of the Indubitability Proof. For the direct target of this argument will be, not that it is impossible to believe the conjunctive proposition, '$Fa \wedge \neg Fa$', but that it is impossible to believe the conjoined propositions separately. That is, it will be impossible to believe Fa and also to believe $\neg Fa$.

In the version of the above section, the Indubitability Proof targets anyone who believes something of the form '$Fa \wedge \neg Fa$'. So right from the start it was directed against belief in conjunctive propositions. This might

[11] Barnes 1969, 306-7, takes PNC as a premise in his rendition of the Indubitability Proof on the strength of 1011b15-20, where it functions in just the way I have suggested, namely, as explaining (12). Because this occurs three chapters later in Gamma 6, and two chapters after Gamma 4's announcement that the proof proceeded 'by means of' PNC, I prefer to keep PNC as an implicit actor in the Indubitability Proof. Of course, nothing of logical substance turns on this nuance.

[12] In the summary of the Appendix, the two ways of getting (19) are separated by a slash, '/', after sentence (19).

[13] Code 1987, 131-32.

appear to clash with Code's claim that belief in a contradiction has as its object two separate beliefs. However, Code is careful to say that Aristotle does not argue 'directly' against the possibility of belief in the conjunctive proposition. I believe we can agree on this. For in introducing (15a) as a premise linking belief in the conjunctive proposition to belief in its conjuncts, I noted that (15a) was 'required' and, hence, not explicitly contained in the text. Strictly, it is the right side of (15a) that is the 'direct' target of the argument, and this argument begins with (16a). The argument against belief in conjunctive propositions relies on the direct argument; but it requires the additional premise, (15a), and so is an extension of the direct argument.

5. *Doxastic Simplification and Property Attribution*

Agreement on structural worries notwithstanding, there is much that is curious about Aristotle's treatment of belief in the Indubitability Proof. Let me begin with (15a), the principle licensing Doxastic Simplification. If Aristotle rejects it, then he risks countenancing the possibility of belief in contradictions, so long as this is belief in the conjunctive proposition. According to this, Heraclitus would be allowed the belief *that water is hot and not hot* but not, simultaneously, both the belief *that water is hot* and the belief *that water is not hot*. Let us express this by saying that Heraclitus could accept the possibility of someone, say *S*, exhibiting, concurrently, the following pattern of beliefs:

20a. $S\ \mathbf{bel}\ (Fa \wedge \neg Fa)$

but not

20b. $(S\ \mathbf{bel}\ Fa \wedge S\ \mathbf{bel}\ \neg Fa)$.

Since anyone can do what *S* can do, Heraclitus would be committed to the general claim

21. $(x)\Diamond(x\ \mathbf{bel}\ (Fa \wedge \neg Fa) \wedge \neg(x\ \mathbf{bel}\ Fa \wedge x\ \mathbf{bel}\ \neg Fa))$,

which says that it's possible that someone, anyone, believe a single contradictory conjunctive proposition without believing, at the same time, the conjoined propositions – namely, the affirmation and its opposed denial (i.e., its negation).

If Aristotle is giving (21) to the opponent of PNC, then the Indubitability Proof is weak tonic at best. Challenged, the opponent can simply insist that he was believing a single contradictory proposition. It is, I think, unlikely that Aristotle would prove so generous a critic. In any case, the claim in (21) is curious for other reasons, and it is these I wish to highlight at the moment.

To grant (21) is to deny Doxastic Simplification, not just for conjunctive contradictory propositions, but for any conjunctive proposition. At least this is so, if the denial is to be a principled denial. But surely it seems that anyone who believes *that Quine is fine and Carnap is smart* is committed to believing *that Quine is fine*. So if Aristotle is denying this, then he will hold something like

21*. $(x)\Diamond(x\ \textbf{bel}\ (Fa \wedge Ga) \wedge \neg(x\ \textbf{bel}\ Fa \wedge x\ \textbf{bel}\ Ga))$.

Now it is hard to know what grounds there might be for holding something like (21*). Some might find a suggestion in Code's remark that to believe a contradiction is to believe two 'separate' propositions. For this suggests that separateness of beliefs may explain why Doxastic Simplification fails.

Thus, grant that the conjunctive proposition, *that Fa and Ga*, can be an object of belief, as well as the separate propositions, *that Fa* and *that Ga*. Although this gives us three distinct objects of belief, how does this fact invite denial of Doxastic Simplification? Well, one might suggest that Property Attribution does the trick. According to Property Attribution, corresponding to each belief there is a doxastic property. Thus, (21*) has a counterpart for properties:

21a. $(x)\Diamond([B{:}(Fa \wedge Ga)]x \wedge \neg([B{:}Fa]x \wedge [B{:}Ga]x))$.

According to (21a) it is possible that someone has the doxastic property, $[B{:}(Fa \wedge Ga)]$, without having the properties, $[B{:}Fa]$and $[B{:}Ga]$. How could this be possible? Well suppose one just insisted that the first property cannot be reduced to the two properties, $[B{:}Fa]$ and $[B{:}Ga]$, plus a compounding operation marked by '$\wedge$'. Rather, it is an irreducible property in its own right, and so a subject can have the property without having the different and distinct property, $[B{:}Fa]$, or without having the different and distinct property, $[B{:}Ga]$. *Were* $[B{:}(Fa \wedge Ga)]$ simply a compound property, it would follow that anyone who had it would also have the compounded parts, $[B{:}Fa]$ and $[B{:}Ga]$. In this case, Doxastic Simplification could not be denied. But on the understanding of Property Attribution that we are currently trying out, $[B{:}(Fa \wedge Ga)]$ is not a compounded property, and, hence, denial of simplification is not blocked. This is not entirely conclusive, however, for we may grant that the doxastic property corresponding to a conjunctive belief is not a mere compound property and still insist that anyone who has that property also has the separate properties in question. Such insistence is acceptable so long as we are willing to say that it is simply a matter of property entailment and that the entailment is primitive and does not rely on 'decomposing' a compounded item. Either way, however, Aristotle ends up with an excessively rich collection of properties, all on equal footing. On both points, I suspect that Aristotle would demur, and so I am inclined to reject this account of complex doxastic properties in favor of the compounded model with its sparser inventory of properties.

This suggests that Aristotle would not reject Doxastic Simplification. Moreover, since property ascriptions, as well as the **bel** operator, are indexed to the same time, it follows on the compounding model that $[B{:}Fa]x$ and $[B{:}Ga]x$ entail $[B{:}(Fa \wedge Ga)]x$; that is, if x has the doxastic property corresponding to Fa and also the doxastic property corresponding to Ga, then x has the doxastic property corresponding to *Fa and Ga*. Hence, belief in 'separate' propositions entails belief in the conjoined proposition. Thus, (15a), to return to the case of interest, could be strengthened to a biconditional. So the claim that the Indubitability Proof provides a direct argument only against holding two separate beliefs (an affirmation and its opposed denial) arguably means just that this is what the argument is explicitly *directed* against.[14] It does not shorten the logical reach of the argument. In particular, it does not compromise the argument's effectiveness against belief in a single conjunctive proposition.[15] So, after all, Heraclitus and friends are not being handled with logical kid gloves.

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[14] It is worth mentioning that the impossibility of believing Fa and believing $\neg Fa$ is not a function of the fact that they are 'separate' beliefs. For this would exclude virtually every pair of propositions as candidates for joint belief, for example, believing *that Quine is fine* and believing *that Carnap is smart*. Rather, only some pairs of separate beliefs resist joint belief, signally, those whose objects are contradictory opposites. It is not entirely clear how we are to understand the separateness of beliefs. It cannot mean that in a given act of believing, the object of belief exhausts the 'doxastic psychological space' of the agent. For then it would be impossible for someone believing Fa to believe anything else, not just $\neg Fa$. So separateness must mean something like having separate existence conditions. Perhaps, this notion can be explicated by appeal to doxastic properties insofar as these are items that may be jointly ascribed to an agent.

[15] I understand this to be consonant with Code's account and, indeed, perhaps an expansion of it.

6. *The Force of the Indubitability Proof*

A more vexing problem concerns the force of the Indubitability Proof.[16] Section 3 raised the question whether the proof is directed against the possibility of believing the negation just of *instances* of PNC, or against the possibility of believing the negation of PNC, the principle itself. It is time to address this more directly. In effect, this is to ask whether the proof supports, in our earlier numbering,

9. $\neg\Diamond(\exists x)(\exists z)(x\ \textbf{bel}\ (Fz \wedge \neg Fz))$,

which I shall christen the 'instantial' reading, or

10. $\neg\Diamond(\exists x)(x\ \textbf{bel}\ \Diamond(\exists z)(Fz \wedge \neg Fz))$,

which I shall christen the 'principled' reading of the conclusion of the Indubitability Proof.

Most commentators take the target of the Indubitability Proof to be the principle, not just its instances. On this principled reading, what is to be shown is the impossibility of believing the negation of PNC itself, the principle, and, presumably, this impossibility extends to instances of the negation. Moreover, there is some reason to think that Aristotle takes his proof to secure the firmness of the principle and so to have established the principled reading, (10). Begin with the observation that *Metaphysics* Gamma 4 appears to continue the discussion of Gamma 3. On the traditional reading, Gamma 4's opening lines address persons who assert that it is possible for something to be and not to be at the same time and that it is possible for some one to believe this.[17] Here the 'opponent' is represented as holding the negation of (10), and so the traditional reading contributes to the impression that this is a shared theme of the two chapters.

The trouble with this, however, is that the Indubitability Proof, as presented in the preceding section, only supports (9), and (9) is not formally sufficient for (10). So Aristotle appears to give an argument that is insufficient for his conclusion. This, of course, assumes that (10) is the intended tar-

[16] Some of the material in this section appears in an earlier version in Wedin 2000.

[17] Thus, Ross 1928 renders the opening lines, 1005b35-1006a2: "There are some who, as we said, both themselves assert that it is possible for the same thing to be and not to be, and say that people can judge this to be the case." He appears to be followed by Kirwan 1971: "There are those who, as we said, both themselves assert that it is possible for the same thing to be and not to be, and [assert that it is possible] to believe so." So also Warrington 1961, who glosses the lines: "i.e., they maintain the possibility of contradiction both in fact and in belief," and Tricot 1974.

get of the Gamma 3 proof. And this, I take it, is the traditional reading, but the traditional reading is not beyond challenge. John Cooper and, independently, David Charles have suggested an alternative reading of 1005b35-1006a2, the opening lines of Gamma 4.[18] They note that Aristotle states only that certain people say (*a*) it is possible for something to be and not to be and also (*b*) ὑπολαμβάνειν οὕτως. That is, these people also hold things to be the case *in this way* (οὕτως), namely, in violation of PNC. But here there is nothing that plausibly refers back to the general denial of PNC itself in (*a*); for this τοῦτο rather than οὕτως would probably be required. So (*b*) refers only to the fact that the theoreticians in question hold something to be the case in agreement with their denial of PNC in (*a*). On this reading, the subject of ὑπολαμβάνειν in the phrase ὑπολαμβάνειν οὕτως need not be ἐνδέχεσθαί εἶναι καὶ μὴ εἶναι, but could be just εἶναι καὶ μὴ εἶναι. So, rather than believing the negation of (10), an admittedly abstract item in any case, Aristotle may have in mind specific statements of the sort allegedly made by Heraclitus and company. And these are just statements that assert particular states of affairs that PNC declares impossible. If this is correct, then the opening of Gamma 4 does not focus on deniers of (10) and so, if Gamma 4 is tracking the themes of Gamma 3, it is not so obvious that Gamma 3's Indubitability Proof targets (10) in the first place. This is not to deny that Gamma 3 holds fast to (10) but only to shift its role in the chapter. It is not the target of *proof*.

On the other hand, at Gamma 4, 1006a4-5, Aristotle does appear to be speaking of the principle itself when he says that it was "shown to be the firmest of all principles." He can only mean that this was shown in the Indubitability Proof of Gamma 3. So there can be little doubt that *Aristotle* thinks that his proof established something about PNC itself. Either Aristotle is guilty of an *ignoratio elenchi* or he must assume that establishing (9), the instantial reading of the conclusion, entitles him to elevate the principle itself as the *firmest* of principles.

There now appear to be at least three ways to regard the Indubitiability Proof: (I) The object of proof is (10) but what is established is only that no instance of the negation of PNC can be believed; (II) the object of proof is not firmness of PNC itself and so not (10), rather PNC is taken to be the firmest principle from the start and a mark of this is the fact,

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[18] In comments on an early version of Wedin 2000, as presented to the 1999 Princeton Classical Philosophy Colloquium.

established by the Indubitability Proof, that no instance of its negation can be believed; (III) the object of proof is (10), and (10) is established by proving that no instance of the negation of PNC can be believed.

On (I), the Indubitability Proof simply fails. This can be highlighted by pressing the point that (9) is not formally sufficient for (10). At best the Indubitability Proof establishes that no instance of the negation of PNC can be believed.[19] It does not establish that the negation of the principle itself cannot be believed. For someone might agree that every proposition he happens to believe is such that he cannot believe it and its negation but nonetheless insist that there might be *some* proposition such that it and its negation can be believed. He is, of course, under no obligation to produce this proposition, for his insistence rests on the general point that claims such as (9) are not formally sufficient to establish claims such as (10). Taking an analogue from standard belief cases, it is plausible that every proposition I believe, I believe to be true, and also that I do not believe that all of my beliefs are true. Here there is no temptation to find me holding contradictory beliefs. Similarly, from the formal point of view, I can hold (9) and also hold that someone could believe that it is possible that there is *some* proposition such that it and its negation are true. But this last belief is just the negation of (10). Hence, to establish (9) is not to establish (10).[20]

Option (II), on the other hand, simply denies that Gamma 3 is concerned to prove anything about the principle, itself, of non-contradiction. Rather, we might imagine, it assumes what other options would prove, namely, that PNC is the firmest of principles, and proceeds to illustrate this by arguing that no instance of the principle's negation can be believed. Hence, we are to think of the Indubitability Proof as focusing on specific, concrete beliefs that appear to run afoul of PNC. Firmness of PNC *itself* is not the object of proof. This option is not without appeal. It

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[19] So, also, by most accounts, for example, Cohen, 1986, 367, and Code 1987, 141.

[20] Although I have appealed to the notion of belief in arguing for the consistency of holding (9) and denying (10), Aristotle's reasoning may be related to a more purely logical point. Rather than the quantified sentence, (9), suppose he is claiming, for indefinite individual propositions, $p_1 \ldots p_n \ldots$, that a certain property fails to hold of each, namely, the property P (= can be believed jointly with its negation). Then we would have Aristotle proving $\neg Pp_1 \wedge \ldots \wedge \neg Pp_n \wedge \ldots$, and this is not inconsistent with holding also that $(\exists p)(Pp)$. Although not inconsistency in the ordinary sense, this looks like what we now call ω-inconsistency. But ω-inconsistency, unappealing though it might be, does not entail ordinary inconsistency and the latter, governed by the principle of non-contradiction, is what preoccupies Aristotle in *Metaphysics* Gamma 3.

spares Aristotle the embarrassment of a failed argument. It also makes for an easy transition to Gamma 4 insofar as Gamma 4 does not attribute to the opponent the claim that it is possible to believe the negation of PNC itself, but rather holds him to assert specific propositions that violate the principle. So it agrees with the Cooper-Charles reading of the opening of that chapter, introduced four paragraphs back. Unfortunately, Gamma 4 also comments that the Indubitability Proof had *shown* PNC to be the firmest of all principles (διὰ τούτου ἐδείξαμεν βεβαιοτάτη αὕτη τῶν ἀρχῶν πασῶν, 1006a4-5). So in Gamma 3 Aristotle cannot have simply *assumed* that it was the firmest principle. Option (II) is incompatible with this.

So we need, I think, some way of connecting the instantial reading of the conclusion of the Indubitability Proof with the claim that PNC is the firmest of principles. Now, of course, *firmness* and *indubitability* are different notions, and so it may be possible to establish the firmness of PNC without arguing, directly, for its indubitability. On this way of thinking, firmness of the *principle* follows from the indubitability of the *instances* of the principle. Recall here that Aristotle's discussion of firmness, detailed in Section 1, proceeds without using the idiom of belief at all. In securing the central thesis, (2), it ties status as firmest to a principle about which error is impossible. The Indubitability Proof is, then, deployed to establish just such immunity to error, and only there does the notion of belief make an appearance. So it may be, as option (III) proposes, that one respectable way to establish something about PNC itself, namely, its immunity to error, and so its firmness, is to prove something about its instances, namely, that their negations cannot be believed.

How does option (III) work? One might suggest that Aristotle simply decrees that a principle is the firmest just in case no negation of an instance of the principle can be believed. Firmness by decree, as we might term this, fails to explain why the instantial reading should have such principled effect. What is needed is some account of how indubitability of instances transfers to indubitability of the principle itself-at least this is so unless we turn a blind eye to the issue that generated our problem (spelled out six paragraphs back in discussing option [I]). Since the transferring in question cannot be a matter of formal entailment, another route must be sought.

Recall that the problem arose because nothing blocked someone's simultaneously holding (9) and denying (10). But this rested on the analogue from standard belief cases, where it is plausible, for example, that every proposition I believe, I believe to be true, and also that I do not believe that all of my beliefs are true. Suppose, however, that standard belief is not the appropriate notion for illuminating the relation between the instantial and the principled readings of the conclusion of the Indubitabilty Proof. Suppose, in short, that the grounds for believing a given proposition are so strong that they guarantee the proposition's truth. One might think of Cartesian grounds in much this way. So I shall call this *C*-based or knowledge-assuring belief. If, of each my *C*-based beliefs, I believe it to be something known *on that basis*, then surely I would deny that any of them could be false. In short, beliefs that allow no space for error to enter are beliefs that assure knowledge.[21] For Aristotle beliefs that are instances of PNC are just such beliefs. Because error is impossible regarding them, belief in such a proposition assures knowledge of it, and it is impossible even to believe their negations.

The point, then, is that *instantial C*-based belief secures *principled C*-based belief. This is not deductive security but a kind of epistemic sanction: we are simply at a loss to account for someone who affirms (9) and denies (10). So far from enhancing the splitting hypothesis, as this might be called, the parallel with *C*-based belief impugns its coherence and, thus, gives principled *force* to the conclusion of the Indubitability Proof. For the parallel now suggests that we are, after all, at a loss to explain someone who affirms the impossibility of anyone's believing a given contradiction but holds that there might be some such instance that someone believed. That is, we are at a loss to explain how anyone complying with (9) could fail to comply with (10). Hence, the former may be said to secure the latter, not deductively, of course, but as a kind of sanction on epistemic credibility.[22] So the parallel with belief, not standard belief but *C*-based or knowledge-assuring belief, may provide the instantial reading with sufficient strength to account for Aristotle's confidence that the Indubitability Proof secures the firmness of the *principle itself* of non-contradiction.[23]

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[21] Here I pass lightly over treacherous ground by simply neglecting questions bearing on the evidential basis of belief and knowledge claims. Fortunately, their resolution has no bearing on the simple point I am making.

[22] Thus, to be clear, I am not claiming that, after all, the Indubitability Proof *deductively* establishes (10) but only that the *force* of establishing (9) is to secure (10). This is possible only because of special features about the objects that (9) ranges over and so goes beyond purely deductive procedures.

[23] An additional point bears comment. Grant that believing *p* on the basis of *C* guarantees knowledge of *p* and grant further that this diminishes the impact of the point

7. *The Reach of the Indubitability Proof*

Suppose that the Indubitability Proof establishes (9), the proposition that it is not possible for someone to believe a contradiction. Still, it is not completely clear how to read this result. Presumably, it covers what might be called transparent contradictions, that is, pairs that consist of a proposition and its express negation. As such, (9) declares it impossible to believe, for example, that the cat is on the mat and the cat is not on the mat, or that water is wet and water is not wet. For these appear to be relatively straightforward instances of the now familiar general formula

9. $\neg\Diamond(\exists x)(\exists z)(x \text{ bel } (Fz \wedge \neg Fz))$.

But what of propositions that are not transparently contradictory but entail, or at least appear to entail, propositions that are transparent contradictions? Are such propositions also impossible for someone to believe?

To clarify the issue, it will be useful to recall that (9) prescribes strong modal medicine, proscribing the very possibility of anyone's believing something of the form '$Fz \wedge \neg Fz$'. Thus, anything entailing impossibilities of the sort covered by (9) will itself be impossible. So, presumably, any belief that has such an entailment will be impossible. What sorts of beliefs might these be? Well, it would seem that if the content of a specimen belief entails a contradiction, then the specimen belief carries commitment to belief in the contradiction. This, at any rate, would be so given

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that there is no formal transition from (9) to (10). Still, one can ask, for what exactly is C the knowledge-assuring basis? For example, is it the basis (*a*) just for a specific proposition p (e.g., *that the orange is round*), (*b*) for all propositions of the same type as p (i.e., any perceptual belief), or (*c*) for any proposition whatsoever? Option (*a*) is too weak because the claim of interest states that there is no *possible* proposition that can be believed along with its negation. So at least (*b*) must be meant. This would mean, perhaps, that for different types of propositions, different bases would have to be spelled out. Of course, I have not tried to do that here; but I need not since option (*b*) is less attractive than option (*c*). This option will be the choice for unitarians, among whom may be counted Cartesians, who regard the criterion of clear and distinct perception as yielding a kind of certainty. Holding that one could not err in believing $\neg(p \wedge \neg p)$ is probably not a function of which (or which type of) proposition p is. So option (*c*) is the most likely way to read basis C. But, again, this is not the place to work out a detailed account, since my aim is merely to remove a reason for supposing that one could coherently affirm (9) and deny (10), in the case of those beliefs involving instances of PNC. For, surely, these will be held to with certainty. Here I am responding to a point pressed by David Charles at the Princeton Colloquium.

22. $(x)(p)(q)((x \text{ bel } p \wedge (p \rightarrow q)) \rightarrow x \text{ bel } q)$,

according to which belief is closed under entailment.

So our question is what to say about propositions that entail something of this form. I shall consider just two kinds of cases here. The first involves propositions, one of which entails the negation of the other only given 'extrinsic' information; the second involves no such information and so I shall say that this sort of entailment is 'intrinsic'. To take the first kind of case, suppose Al believes that the author of *Silas Marner* is smart but also that Mary Ann Evans, being a woman, is not smart. There seems nothing logically odd in maintaining both of these beliefs. But, of course, Al's misguided views notwithstanding, Mary Ann Evans is the author in question, under the pseudonym of George Eliot. With this extrinsic information, it might appear to follow that Al believes that the author of *Silas Marner* is smart and not smart. Or suppose, to abuse an example of Aristotle's, that Al believes that the man in the white tunic is not his father. Given the extrinsic information that the tunicated gentleman is in fact his father, some might conclude that Al believes his father is not his father.

Were these cases accepted at face value, it would follow by (9), the 'conclusion' of the Indubitabilty Proof, and the principle of doxastic closure registered in (22), that it is not merely false but impossible for Al to believe that the author of *Silas Marner* is smart and that Mary Ann Evans is not, or to believe that the man in the white tunic is not his father. Surely, this is nothing Aristotle would want to embrace. Now we could reply to this situation by allowing that Al does not believe *de dicto*, but does believe *de re*, that his father is not his father, and so on. Then the Indubitability Proof could be extended only on the basis of *de dicto* believings, that is, beliefs that one knowingly holds as such. However, as we shall shortly see, Aristotle's style of indicting the views of some of his predecessors suggests that he may not find this reply appealing.

For the moment it will be enough to remark that because extrinsic information is not available within the relevant doxastic context, it cannot be used to generate *belief* commitments of the sort we have just sketched. So it is very unlikely that Aristotle would extend (9), and the Indubitability Proof, to the sorts of beliefs here awarded to Al. This is obviously correct, for there is nothing about such beliefs themselves that raises suspicions concerning the possibility holding them jointly. Only with the addition of extrinsic information can we even begin to suppose that Al shuffles into logical darkness. With respect to principle (22), this is to say that *p* itself is not sufficient for the entailment in question, but requires the truth of additional propositions lying outside the doxastic space of the believer in question. Of course, *were* these propositions also believed by Al, they would cease to be extrinsic and so might well commit him to believing the entailed contradictions. This suggests that we may attribute *belief* in a contradiction to an agent only on the basis of propositions he actually owns up to.

With this we are brought to the second kind of case, namely, where a belief, or set of beliefs, 'intrinsically' entails a contradiction and so commits the agent to believing a contradiction. To say that a first proposition intrinsically entails a second is at least to say that the second can be gotten from the first by reasoning alone. Sometimes this occurs immediately, as when *a's being colored* can be gotten from *a's being red*. One might say that simply understanding what *red* is, is sufficient for the inference. So if someone believes that something is red, then we can take it that he believes the thing is colored; or, since the terrain of belief ascription is notoriously treacherous, we can at least say that he is committed to the belief that the thing is colored. And, presumably, even Al can be persuaded of this by a quite simple piece of reasoning. In like manner Heraclitus's asseveration that seawater is most pure and most polluted yields by equally simple reasoning that seawater is most pure and not most pure. So holding to the asseveration would entail belief in an instance of the denial of PNC.[24]

This sobering conclusion can, however, result from more complex patterns of reasoning. Indeed, Aristotle ascribes just such patterns to the most central views of a number of his predecessors. That is, he finds them committed to denial of PNC by a number of their favored theses. Sometimes he suggests that this is done with full awareness, as in *Metaphysics* Gamma 5, where he reports that because certain of the writers on nature held that contraries emerge from the same thing and that what is not cannot come to be, they asserted that the same thing was all along both so-and-so and not so-and-so. Sometimes denial of PNC is held to be an implicit commitment, as in *Metaphysics* Gamma 4, where the thesis of Protagoras, that what appears to be the case is the case, involves denial of PNC. Then, later, in Gamma 5, a set of general theses about change implicate so-called Heracliteans in denying the principle of non-contradiction.[25]

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[24] Here I put aside whether such an unnuanced reading is fair to Heraclitus.

[25] Indeed, in strongly denying it, that is, in asserting the contrary of PNC, namely,

In all these cases, the entailment is intrinsic because it is established simply by reasoning from given theoretical theses to the denial of PNC. As such, they fit the prescription that where p entails q, if someone believes p, then that person believes q; so intrinsic entailment is the notion needed for (22), the closure of entailment under belief. What exactly would this show? To take a case in point, Aristotle claims that the thesis of Protagoras, that what appears to be the case is the case (PT), entails denial of PNC, indeed, that it involves the strong denial of PNC, namely, the claim that every thing has every property and does not have it. Informally, this may be represented as

23. ((PT $\rightarrow$ every x has F and does not have F) $\wedge$ a **bel** PT) $\rightarrow$ a **bel** every x has F and does not have F.

Assume that the *reasoning* covering the first arrow, i.e., the reasoning from 'PT' to 'every x has F and does not have F', is unexceptionable.[26] Then, because the Indubitability Proof declares that it is impossible for a to believe a contradiction, the consequent of (23) is false, and so 'a **bel** PT' must be false. But the consequent in question is not merely false. It is impossible, and because the reasoning is unexceptionable, the source of the impossibility must lie with PT. In particular, it must lie in a's believing PT. Hence, this would appear to be an impossible object of belief.

Admittedly, Aristotle does not raise this issue, but it is hard to see how he could avoid it.[27] So our question is whether belief-to-belief entailments of this sort undermine his attack on Protagoras, to take the case at hand, not to mention other predecessors whose theoretical commitments are said to entail denial, in one way or another, of the principle of non-contradiction. I think not, for Aristotle would insist, I submit, that despite professing to believe PT, once presented with the reasoning in question, the opponent will be forced to concede that he only thought he could believe such a thing. What he has been shown is that this is in fact impossible because it entails something that Aristotle has *proven* impossible, namely, believing the denial of PNC or an instance of the denial. Because PT is connected to strong denial by a purely rational train of reasoning, the

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that everything has every property and does not have it. I discuss this in Wedin (2004).

[26] Aristotle produces an argument for the entailment. See Wedin 2003.

[27] Because it is hard to see how he could deny the principle: $(p \rightarrow q) \wedge \neg \Diamond q \rightarrow \neg\Diamond p$. One might deny that '$a$ **bel** PT' is a proper substitution instance for 'p', but this would need argument.

connection in no way depends on 'extrinsic' facts. Thus, unlike the cases involving Al discussed several paragraphs back, here it is plausible to press the impossibility of believing PT. For there are no facts or circumstances whose emergence could shield the offending belief from the entailment because the entailment is forthcoming from the content of the belief, albeit by way of argument.[28]

But now there arises a difficult question. When Protagoras believes, or claims to believe, PT we may presume that he has some doxastic property or that he does not. If he does not, then it is unclear that we are entitled to credit him with any belief at all – at least by Aristotle's lights. If he has such a property, which one is it? Surely, it would not be the property he *would* have *were* he to believe a contradiction straightaway. Setting aside the Indubitability Proof's declaration that this is impossible, this property would give him a belief that, on the face of it, looks quite different from the belief that what appears to be the case is the case. Now, of course, contradictions are not the only impossible propositions; so belief in PT may be impossible without being itself a contradiction. And, indeed, Aristotle says only that the friend of PT *would* have to believe contradictions and so *would* have to have contradictory doxastic properties.[29] Thus, PT is not faulted for directly requiring possession of such properties but for implying something that does. Still, Aristotle's argument seems

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[28] Of course, this assumes that Aristotle's opponent is open to reasoning. This appears to be correct, for when Aristotle turns to defense of PNC itself, in Gamma 4, his argument, the notorious 'elenctic' proof, presupposes that the opponent welcomes, within certain bounds, the effects of deductive reasoning. I discuss this feature of the elenctic proof in Wedin 2000, especially Section 5.

[29] Here it is important to register the difference between merely entertaining a proposition and actually believing it. For Aristotle must be able to claim that it is impossible to believe a contradiction and, in doing so, not have committed himself, in any sense, to contradictions as an object of belief. In the general case, for someone to believe that p, it must be possible that the world be as p declares and, also, that he have the doxastic property corresponding to his belief that p, namely, $[B{:}p]$, in our idiom. So the reformed theorist, who renounces belief in instances of the negation of PNC, need only believe$_2$ that *were* he to believe$_1$ that $Fa \wedge \neg Fa$, then the world *would* have to be such as to contain the contradictory state of affairs, Fa and $\neg Fa$ and he would have to have the corresponding contrary doxastic properties $[B{:}Fa]$ and $[B{:}Fa]^*$. Clearly, Aristotle, not to mention his reformed theorist, must be able to hold the complex belief introduced by 'believe$_2$'. This, in turn, gives him a complex doxastic property corresponding to the content of what is believed$_2$, but it does not him commit him to believing, as opposed to merely entertaining, the proposition that Fa *and* $\neg Fa$, and, hence, it does not ascribe to him the contrary doxastic properties $[B{:}Fa]$ and $[B{:}Fa]^*$.

to require that it is impossible for anyone to believe PT. This, in turn, calls for some account of what the advocate of PT takes himself to be believing or, at the very least, some account of his alleged doxastic activity. Is he merely mouthing words, to appropriate Aristotle's idiom, or is the Protagorean indulging in some deeper kind of mental error? Unfortunately, Aristotle says very little about this, and so we are left to speculate.

What is, however, clear in the above account is the *explanatory* role accorded PNC. For what makes the belief in PT impossible is the fact that belief in the negation of PNC, or an instance of the negation, is impossible because it itself requires that a contradictory state of affairs obtain. Thus, the latter impossibility, the impossibility of believing an instance of the negation of PNC, explains the former impossibility, the impossibility of believing PT. So it is, perhaps, unsurprising that the explanatory status of PNC looms large in the final lines of Gamma 3, where it is declared the ultimate principle.

8. *On the Ultimacy of PNC*

Aristotle closes *Metaphysics* Gamma 3 by claiming that PNC enjoys some sort of ultimacy as a demonstrative principle.[30] Having established to his satisfaction that it is impossible to believe denials of PNC, he says:

> That is why all those who demonstrate go back to this doctrine (δόξα) in the end; it is in the nature of things the principle of all the other axioms also (φύσει γὰρ ἀρχὴ καὶ τῶν ἄλλων ἀξιωμάτων αὕτη πάντων) (1005b32-34).

This closing flourish to the chapter has proven troubling. Indeed, by most accounts it is unclear why Aristotle even includes the lines. After all, if the Indubitability Proof holds, it would establish that the negation of PNC, or instances of the negation, cannot be believed, and, so, establish that PNC enjoys an elevated kind of firmness. But Aristotle wants more, for he claims that PNC is *the* firmest principle of all. Of course, traditionalists might insist, a tract on being should pinpoint the highest principle of being, and one might suppose that this is what motivates Aristotle's claim. Nonetheless, it remains a fact that the Indubitabilty Proof gives no reason to suppose that PNC is firmer than all other principles, for a number of these enjoy the same immunity to error.

[30] This section draws, with modifications, on Wedin 2000.

On my view, the ultimacy claim is not merely a closing flourish in the cause of high metaphysics, but effectively completes Aristotle's argument. In particular, it does not merely assert that PNC is the firmest principle, but also suggests a reason for this. And the reason will explain the singularity of PNC as the firmest principle. Of course, this presumes that the ultimacy claim holds up – something that has been challenged by more than one scholar. So I need to consider the chief objection to the claim. Doing so will also help show how PNC's ultimacy gives it standing as *the* firmest principle.

Lukasiewicz (1910b) reacted strongly to Aristotle's closing flourish to Gamma 3 and urged rejection of the view that PNC is the highest principle of all demonstrations. Although this holds for indirect proofs, he maintained that it is false for direct proofs and that in general any number of logical principles are independent of PNC. This opinion may lurk in the background of Kirwan's complaint that immunity to disbelief does not establish that every argument relies on PNC but only that no argument questions it.[31]

The complaint represents Aristotle as having failed, and failed badly, in assessing the logical station of his firmest principle. The issue can be sharpened by tracking Lukasiewicz's account of Aristotle's troubles. Direct proofs do not, but indirect proofs do, presuppose PNC. In a direct proof we may have, for example, that, given $p \rightarrow q$, and p, we are to infer q. The idea is that this is one of "innumerable deductions . . . which proceed only by affirmative propositions; consequently, the principle of contradiction finds no application to these because it always joins an affirmative proposition and its contradictory negative."[32] This sounds plausible, but does it stand scrutiny?

Well, it is quite correct that direct inferences do not typically *use* PNC as part of the reasoning. This, however, does not establish that such inferences are independent of PNC because what is presupposed by a pattern of inferential reasoning need not be a part of that reasoning itself. So let us take Aristotle at his word when he says that in the end all demonstrating goes back to PNC, and let us suppose, further, that the reasoning is deductive. Then he is claiming that all deductive reasoning somehow goes back to PNC. Arguably, this calls for a connection between patterns of reasoning and PNC. To take a case in point, then, the above pattern of reasoning depends on

[31] Kirwan, 1971, 90.
[32] Lukasiewicz 1910b, 504.

24. $((p \rightarrow q) \wedge p) \rightarrow q$,

in particular on its validity. The outer parenthesized schema may be said to *imply* q. Thus, the conjunction of the antecedent with the negation of the consequent should lead to an inconsistency.[33] In the case of (24), we would have on the left: $\neg(p \wedge \neg q) \wedge p$; and on the right: $\neg q$. But the left side is equivalent to $(\neg p \vee q) \neg p$ and, hence, to q. So, we are left with q and $\neg q$. We, thus, confirm that (24) is valid. More to the point, however, we do so by appeal to the principle of non-contradiction. Hence, one can conclude that the validity of (24) depends on the principle of non-contradiction, even if no application of (24) or instances of (24) *uses* the principle. The same result is yielded by any pattern of reasoning that is deductively valid.[34]

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[33] With Quine1966, 100: "One schema implies another if and only if the one in conjunction with the other's negation is inconsistent."

[34] Thus, simplification depends on the validity of (25), $(p \wedge q) \rightarrow p$, for conjoining $p \wedge q$ with $\neg p$ leaves us with $p \wedge \neg p$, which again violates the principle of non-contradiction. Why, then, does Lukasiewicz insist that indirect arguments alone presuppose PNC? We are already familiar with his claim that because (24) and (25) contain only affirmative propositions there could be no dependence on PNC. But, as we have just seen, this does not detract from the fact that their *validity* can be seen to rest on the principle. Perhaps, however, Lukasiewicz has something else in mind. If so, it should be evident from his account of how indirect proofs *do* depend on PNC. What he says is this (1910b, 499): "The *ad impossibile* mode of inference turns namely on the principle of contraposition which – as symbolic logic has shown – presupposes the principle of contradiction. This can also be put into words: The *ad impossibile* mode of inference runs: If a is, then b must be; now b is not; thus, a also cannot be. Reason: Were a to be, then a contradiction would ensue, for b must be, which it is not." In more customary terms, given $p \rightarrow q$, and $\neg q$, we are to infer $\neg p$ *because* were we to have p, we would have q and so we would have q and $\neg q$. But this violates PNC and so PNC is presupposed by indirect proofs. Notice that Lukasiewcz might have arrived at this result by asking what would be the case were the mode of inference not deductively valid. Thus, were (26), $((p \rightarrow q) \wedge \neg q) \rightarrow \neg p$, not valid, we would have on the left side of the arrow $(\neg p \vee q) \wedge \neg q$, which simplifies to $\neg p$, and on the right we would have p. Once again, the principle of non-contradiction is violated and, so, we may conclude that (26) is valid. In any case, the result shows, Lukasiewicz avers, that indirect proofs depend on PNC. For some reason he thinks this case is different from cases (24) and (25) and from direct patterns of inference generally. But it is hard to see the force of this. No more than those cases, do applications of (26) or instances of (26) *use* PNC. (26)'s dependence on PNC is secured by asking why the inference goes through and answering that, were it not to go through, a contradiction would result. This is precisely the procedure we followed in assessing (24) and (25) – *modus ponens* and simplification. So in this sense of 'presupposition' (26) presupposes PNC, and, in that same sense, (24) and (25) presuppose the principle. Of course, there may

So, arguably, there is a sense in which PNC is the doctrine that everyone who demonstrates goes back to in the end – not as the principle *from which* all deductions start, in which case it would be used in all deductions, but rather as a presupposition of the *validity* of the principles that are used in such deductions, namely, the principles of deductive reasoning. In this way PNC's claim to ultimacy holds despite the fact that it is not *used* in all cases of deductive reasoning.

There remains a worry. If PNC is such a presupposition, then is it not parading as a principle that is somehow 'deeper' than other logical principles? This, of course, will be challenged on the grounds that the validity of principles such as $p \wedge q \rightarrow p$ or $p \rightarrow p$ is hardly less transparent than that of $\neg(p \wedge \neg p)$. Nonetheless, there is a reason Aristotle gives pride of place to PNC. Recall his claim that it is *because* PNC is the firmest of principles that it is the principle every demonstration goes back to. From this point of view, we may take the principle not as *establishing* the validity of principles of deduction but rather as *displaying* their deductive firmness. Someone might suppose it possible to grant their deductive utility, even validity, but still insist that they are not immune to error, that is, that someone might be mistaken about them. This, however, requires that it be possible for principles of demonstration not to hold; and this, in turn, amounts to the possibility that PNC fail to hold. Not only is this impossible, by Gamma 3's Indubitability Proof it cannot even be believed. Hence, the firmness attaching to PNC is inherited by all principles whose denials flout the principle of non-contradiction.[35] Because these principles inherit their firmness *from* PNC and because PNC establishes its *own* firmness, he declares that it is the principle of all other principles.[36] Thanks to its role

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be another sense of presupposition according to which indirect, but not direct, proofs presuppose PNC.

[35] To take a case discussed in the above footnote, (25) is not demonstratively firm if it is possible that some x believes $\neg[(p \wedge q) \rightarrow p]$. But this requires that it be possible that x believe $(p \wedge \neg p)$, something the Gamma 3 Indubitability Proof shows to be impossible. So (25) is also immune to error and, hence, is demonstratively firm.

[36] So far, I have taken Aristotle's claim, that PNC is the doctrine all demonstration goes back to, as a claim about propositional connections. But the closing flourish goes on to claim that it is the principle of all other axioms. This appears to imply, or at least suggest, that PNC is itself an axiom, and it invites the thought that it may be the principle of axioms other than those figuring in demonstration. In any event, my account should cover certain principles that are not standard principles of inference, that is, those that do not govern relations between propositions. One of Aristotle's favorites is the so-called 'equals axiom': where $A = B$, $(A + C = D) \rightarrow (B + C = D)$. According to my account, then, if x believes the axiom does not hold, then x believes

in *explaining* the firmness of other principles, PNC can be declared *the* firmest principle of all. Thus, the ultimacy claim completes the argument in favor of the singular status of the principle of non-contradiction. So far from being merely *one* of the firmest principles, it assures that PNC is *the* firmest principle – just as Aristotle promised.[37]

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it is possible to have $(A + C \wedge D) \wedge (B + C \wedge \neg D)$, and, thus, that it is possible to have $D \wedge \neg D$. That is, given $A = B$, it must be possible for the same thing to sum to D and not to sum to D. But this appears to violate PNC. So, again, x can deny the equals axiom only on pain of contradiction. Although only analogous to the proposition-friendly principles, (24), (25), and (26), the analogy is close enough, I think, to explain why Aristotle might have regarded PNC as presupposed by this sort of axiom as well.

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APPENDIX

SUMMARY OF THE INDUBITABILITY PROOF

11.	$\neg\Diamond(\exists x)(Fx \wedge \neg Fx)$	P
12.	$\neg\Diamond(\exists x)(Fx \wedge F^*x)$	11,13
13.	$(x)(F^*x \rightarrow \neg Fx)$	P
14.	$(x)(x$ *believes* Fa is contrary to x *believes* $\neg Fa)$	P
14b.	$(x)(x\ \mathbf{bel}\ \neg Fa \rightarrow [B{:}Fa]^*x)$	14, Property Attribution
14a.	$(x)(x\ \mathbf{bel}\ Fa \rightarrow [B{:}Fa]x)$	14, Property Attribution
15.	$(x)(x\ \mathbf{bel}\ (p \wedge q) \rightarrow x\ \mathbf{bel}\ p \wedge x\ \mathbf{bel}\ q)$	P
15a.	$(x)(x\ \mathbf{bel}\ (Fa \wedge \neg Fa) \rightarrow x\ \mathbf{bel}\ Fa \wedge x\ \mathbf{bel}\ \neg Fa)$	15
16a.	$(x)(x\ \mathbf{bel}\ (Fa \wedge \neg Fa) \rightarrow [B{:}Fa]x)$	14a,15a
16b.	$(x)(x\ \mathbf{bel}\ (Fa \wedge \neg Fa) \rightarrow [B{:}Fa]^*x)$	14b, 15a
17.	$(x)(x\ \mathbf{bel}\ (Fa \wedge \neg Fa) \rightarrow [B{:}Fa]x \wedge [B{:}Fa]^*x)$	16a, 16b
18.	$(x)(x\ \mathbf{bel}\ (Fa \wedge \neg Fa) \rightarrow [B{:}Fa]x \wedge \neg[B{:}Fa]x)$	13,17
19.	$(x)\neg(x\ \mathbf{bel}\ (Fa \wedge \neg Fa))$	11,18 / 12,17
19a.	$\neg(\exists x)(x\ \mathbf{bel}\ (Fa \wedge \neg Fa))$	19

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